8).

(a) ~~State~~ Nyquist Sampling theorem: A continuous time signal can be represented in its samples and can be recovered back when sampling frequency f_s is greater than or equal to the twice the highest frequency component of message signal, i.e:

$$f_s \geq 2 f_m$$

$$x_a(t) = 5\cos 100\pi t + 10 \sin 300\pi t + \sin 150\pi t$$

$f_1 = 50$, $f_2 = 150\ Hz$, $f_3 = 75\ Hz$

$f_m = \max(f_1, f_2, f_3) = f_2 = 150\ Hz$.

Nyquist rate, $f_s = 2 f_m = 300\ Hz$.

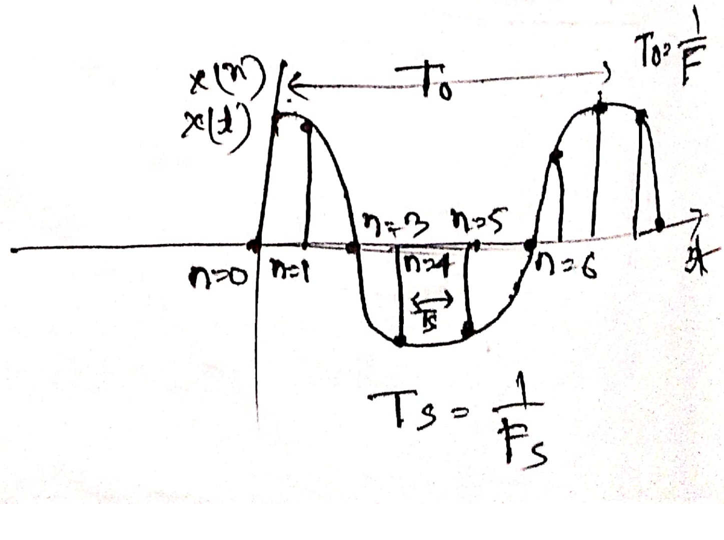
(b) When an analog signal $x_a(t)$ is sampled periodically at a sampling interval T, the resulting discrete-time signal is given by,

$$x[n] = x_a(nT), \quad n \in \mathbb{Z}$$

Here, $T = \frac{1}{F_s}$, where F_s is a sampling frequency.

For an sinusoidal signal,

$$X_a(t) = A\cos(2\pi F t + \phi)$$



The sampled signal becomes,

$$x[n] = A\cos(2\pi F nT + \phi)$$

Now, $x[n] = A\cos\left(2\pi \cdot F \cdot n \frac{1}{F_s} + \phi\right)$ $\left[T = F_s^{-1}\right]$

$= A\cos\left(2\pi \cdot \frac{F}{F_s} \cdot n + \phi\right)$

Let, $f = \frac{F}{F_s}$, where f is the normalized digital frequency.

$x[n] = A\cos(2\pi \cdot f \cdot n + \phi)$.

This shows the relationship ~~b/w~~ between analog and digital frequency with sampling frequency.

$$f_{digital} = \frac{F_{analog}}{F_{sampling}}$$

F_{analog} : $-\infty$ to ∞ (Theoretically)

Nyquist :

$-\frac{F_s}{2} < F_{analog} < \frac{F_s}{2}$

$-\frac{1}{2} < f_{digital} < \frac{1}{2}$

Angular Frequency : $\to \omega T$

$x[n] = A\cos(\Omega n + \phi)$

$x(t) = A\cos(\omega t + \phi)$; $\left[-\pi < \omega < \pi\right]$

Digital angular frequency.

$\Omega = \omega T$

$\Rightarrow$ ~~$2\pi F \cdot T = \omega T$~~

$\Rightarrow \Omega = 2\pi \frac{F}{F_s}$

So, the digital (angular) frequency is just 2π times ~~x~~ of the the frequency analog signal F normalized to the sampling rate F_s.