CT-1

1	Define differential equation, ordinary and partial differential equation with examples?	3
2	Form the differential	

Ans :

Differential Equation: An equation involving derivatives or differentials of one or more dependent variables with respect to one or more independent variables is called differential equation.

$$\frac{dy}{dx} = \sin x + x \quad \cdots\cdots (i)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \cdots\cdots (ii)$$

$$\frac{d^4 y}{dx^4} + \frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^5 = e^x \quad \cdots\cdots (iii)$$

Ordinary Differential Equation: A differential equation involving derivatives w.r.t only a single independent variable is called ordinary differential equation.

$$\frac{dy}{dx} = \sin x + x \quad \cdots\cdots (i)$$

$$\frac{d^2 y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0 \quad \cdots\cdots (ii)$$

Partial Differential Equation: A differential equation involving derivatives w.r.t more than one independent variable is known as partial differential equation.

$$\frac{\partial^2 v}{\partial t^2} = k\left(\frac{\partial^3 v}{\partial x^3}\right)^2 \quad \cdots\cdots (i)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \quad \cdots\cdots (ii)$$

2	Form the differential equations for the following: (i) $y = Ae^{2x} + Be^{-2x}$ (ii) $y = A\cos nt + b\sin nt$, where A and B being arbitrary constants.
3	What is Wronskian? Find the W...

(i) $y = Ae^{2x} + Be^{-2x}$

$\Rightarrow$ Given that,

$$y = Ae^{2x} + Be^{-2x}$$

$$\Rightarrow y' = 2Ae^{2x} - 2Be^{-2x} \quad [\text{Diff. w.r.t. } x]$$

$$\Rightarrow y'' = 4Ae^{2x} + 4Be^{-2x} \quad [\text{Diff. w.r.t. } x]$$

$$\Rightarrow y'' = 4(Ae^{2x} + Be^{-2x})$$

$$\Rightarrow y'' = 4y$$

$$\Rightarrow y'' - 4y = 0.$$

(ii) $y = A\cos nt + b\sin(nt)$

$\Rightarrow$ Given that,

$$y = A\cos nt + b\sin(nt)$$

$$\Rightarrow y' = -nA\sin(nt) + nb\cos(nt) \quad [\text{Diff. w.r.t. } x]$$

$$\Rightarrow y'' = -n^2 A\cos(nt) - n^2 b\sin(nt) \quad [\text{Diff. w.r.t. } x]$$

$$\Rightarrow y'' = -n^2 (A\cos nt + b\sin nt)$$

$$\Rightarrow y'' = -n^2 y$$

$$\Rightarrow y'' + n^2 y = 0$$

	..., where A and B being arbitrary constants.	
3	What is Wronskian? Find the Wronskian of e^{x}, e^{-x} and e^{2x}. hence, conclude whether or not these are linearly independent.	5

Ans: Wronskian: The wronskian of a function $y_1(x), y_2(x), \ldots, y_n(x)$ is denoted by $W(x)$ and defined to be the determinant.

$$W(x) = \begin{vmatrix} y_1(x) & y_2(x) & \cdots & y_n(x) \\ \vdots & & & \\ y_1^{n-1}(x) & y_2^{n-2}(x) & \cdots & y_n^{(n-1)}(x) \end{vmatrix}$$

Given that,

$$y_1 = e^{x} \qquad y_2 = e^{-x} \qquad y_3 = e^{2x}$$

$$\Rightarrow y_1' = e^{x} \qquad \Rightarrow y_2' = -e^{-x} \qquad \Rightarrow y_3' = 2e^{2x} \quad [\text{Diff. w.r.t } x]$$

$$\Rightarrow y_1'' = e^{x} \qquad \Rightarrow y_3'' = e^{x} \qquad \Rightarrow y_3'' = 4e^{2x} \quad [\text{Diff. w.r.t } x]$$

So, the wronskian of the function,

$$W(x) = \begin{vmatrix} e^{x} & e^{-x} & e^{2x} \\ e^{x} & -e^{-x} & 2e^{2x} \\ e^{x} & e^{-x} & 4e^{2x} \end{vmatrix}$$

$$= \begin{vmatrix} 0 & e^{-x}+e^{-x} & e^{2x}-2e^{2x} \\ 0 & -e^{-x}-e^{-x} & 2e^{2x}-4e^{2x} \\ 1 & e^{-x} & 4e^{2x} \end{vmatrix} \quad \begin{bmatrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \end{bmatrix}$$

$$= 1\left[2e^{-x}\times(-4e^{2x}) - (-2e^{-x})\times(-e^{2x})\right]$$

$$= 8\times e^{-x}.e^{2x} - 2e^{-x}.e^{2x}$$

$$\Rightarrow 6e^{-x}.e^{2x}$$

$W(x) \neq 0$, these are linearly independent.

Ans:

Order : The highest order derivatives involved in a differential equation is called Order of the Differential Equation.

$$\frac{dy}{dx} = \sin x + x \rightarrow \text{Order } 1 \text{ (first order)}$$

$$\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^5 = e^x \rightarrow 4^{th} \text{ order}$$

Degree: The degree or power of the highest order derivative which occurs in the *polynomial* differential equation.
(The differential equation must be a polynomial equation in derivatives for the degree to be defined.)

$$\frac{dy}{dx} = \sin x + x \rightarrow \text{degree } 1$$

$$\left(\frac{d^4y}{dx^4}\right)^3 + \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^5 = e^x \rightarrow \text{degree } 3$$

Linear Differential Equation: A differential equation is called linear if

- Every dependent variable and every derivative involve occurs in first **degree** only
- No product of **dependent** variables and derivative occur

Example:
xdy/dx+2y = x^2
dx/dy – x/y = 2y
dy/dx + ycot x = 2x^2

Non-linear Differential Equation: A differential equation which is not linear is called a non-linear differential equation.
Example:

$$\frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{3/2} \quad \ldots\ldots\ldots (i)$$

5	Find the order and degree of the following differential equations. Also classify them as linear and non-linear: (i) $y = \sqrt{x}\left(\frac{dy}{dx}\right) + \frac{k}{\left(\frac{dy}{dx}\right)}$ (ii) $y = x(dy/dx) + a\left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^{1/2}$	6

Ans:

(i)

Given that,

$$y = \sqrt{x}\left(\frac{dy}{dx}\right) + \frac{K}{\left(\frac{dy}{dx}\right)} \quad \ldots (i)$$

The highest derivatives occurs in (i) is one. So it is first order DE.

From (i) →,

$$\Rightarrow y = \sqrt{x}\left(\frac{dy}{dx}\right) + \frac{K}{\left(\frac{dy}{dx}\right)}$$

$$\cancel{\Rightarrow y = \sqrt{x}}$$

$$\Rightarrow y\left(\frac{dy}{dx}\right) = \sqrt{x}\cdot\left(\frac{dy}{dx}\right)^2 + K \quad \ldots (ii)$$

The highest order's degree is 2. So, the degree of the DE is 2.

— Non linear DE.

$$(ii) \quad y = x\cdot\frac{dy}{dx} + a\left\{1 + \left(\frac{dy}{dx}\right)\right\}^{1/2} \quad \ldots (i)$$

$$\Rightarrow y - x\frac{dy}{dx} = a\left\{1 + \left(\frac{dy}{dx}\right)\right\}^{1/2}$$

$$\Rightarrow \left(y - x\frac{dy}{dx}\right)^2 = a^2\left(1 + \frac{dy}{dx}\right) \quad \text{[Squaring both side]}$$

$$\Rightarrow y^2 - 2xy\cdot\frac{dy}{dx} + \left(x\frac{dy}{dx}\right)^2 \cancel{- a^2} \cancel{+ x^2 (1} = a^2\left(1 + \frac{dy}{dx}\right)$$

Order : 01

Degree : 02

Non-linear DE.