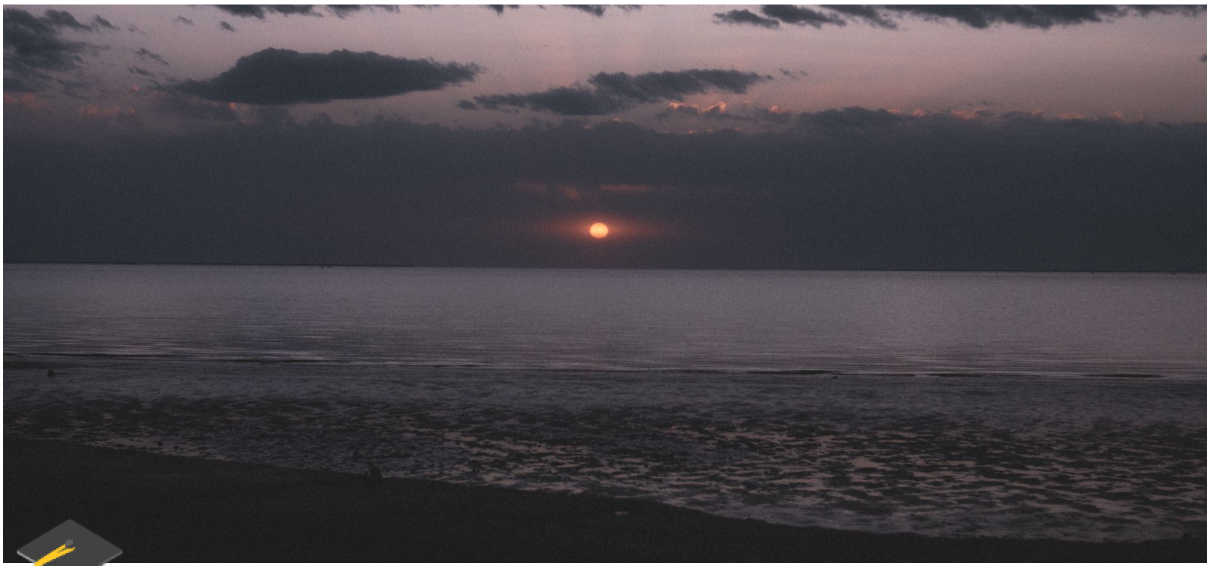

Question Analysis

Created	@January 10, 2023 9:34 PM
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Email	

Numerical Analysis:

~~Newton forward Interpolation(proof + math)*****~~

~~forward difference operator*~~

~~construct forward difference table**~~

~~Lagrange interpolation ****~~

~~Newton divided formula *~~

~~Euler (math + description+derivation)***~~

~~Modified Euler~~ (how it improves accuracy)*

~~Taylor ** (skipped)~~

~~Range CUTE-AAAA**~~

~~absolute error+relative error**, overflow and underflow*~~

~~find percentage of error math*~~

~~chopping, general equation of chopping*~~

~~find absolute and relative error*~~

~~Numerical Differentiation * (skipped)~~

~~simson 1/3 ****~~

~~trapezoidal rule(def^n + math)***~~

~~Bisection (description, explanation, proof) ***~~

~~Newton raphson (math + equation)****~~

~~trancendental equation and characteristics (*)~~

~~Gauss method (describe)*~~

~~LU decomposition **~~

~~Curve(math + diff)**~~

numerical integration (proof+defn)* * (skipped)

~~find first and second derivative ***~~

~~iretative method*~~

~~gauss-seidel method*~~

gauss-jordan*

~~taxonomoy of error*~~

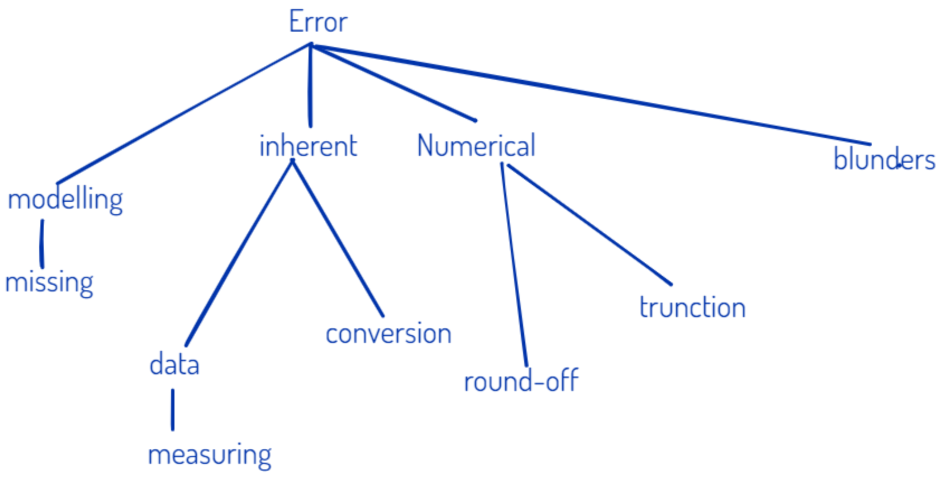
~~shifting operator basic~~

Errors:

Error, in applied mathematics, **the difference between a true value and an estimate, or approximation, of that value**

.

Taxonomy of error:



***truncation**

Absolute Error

Absolute error is the difference between measured and the actual value of a quantity.

Keyword: measured value - actual value

If x is the actual value of a quantity and x0 is the measured value of the quantity, then the absolute error value can be calculated using the formula

$\Delta x = |x0-x|$.

Here, Δx is called an absolute error.

For example, 24.13 is the actual value of a quantity and 25.09 is the measure or inferred value, then the absolute error will be:

Absolute Error = |25.09 – 24.13|
= 0.86

Relative Error

The relative error is defined as the ratio of the absolute error of the measurement to the actual measurement.

Keyword: absolute error/actual error

If x is the actual value of a quantity, x0 is the measured value of the quantity and Δx is the absolute error, then the relative error can be measured using the below formula.

Relative error = (x0-x)/x = (Δx)/x

Rounding Error

Rounding error is the difference between a rounded-off numerical value and the actual value.

As an example of rounding error, consider the speed of light in a vacuum. The official value is 299,792,458 meters per second. In scientific (power-of-10) notation, that quantity is expressed as 2.99792458 x 108. Rounding it to three decimal places yields 2.998 x 108. The rounding error is the difference between the actual value and the rounded value, in this case (2.998 - 2.99792458) x 108, which works out to 0.00007542 x 108
. Expressed in the correct scientific notation format, that value is 7.542 x 103.

Rounding error = |rounded-off numerical value - actual value|

Percentage of Error

To see how the calculation works, let's look at a quick example.

While measuring the layout for a pool, a landscaper accidentally records 8m. What is the percentage error if the actual length is 10m?

To solve for this, we'll use the formula:

Percentage Error = ((Estimated Number – Actual Number)/ Actual number) x 100

- Where the Actual Value = 10m
- And the estimated value = 8m.

Step 1. Subtract the actual value from the estimated value.

8m – 10m = -2m

Step 2. Divide the results with the actual value

-2m/10m = -0.2

Step 3. To find the percentage error, multiply the results by 100

-0.2 x 100 = -20%

The percentage error in the measurement was -20%

Percentage Error = 8 – 10/10 x 100 = -2/10 x 100 = -20%.

(Take absolute value)

Summary:

- **Absolute Error = |Experimental Measurement – Actual Measurement|**
- **Relative Error= Absolute Error/Actual Measurement**
- **Percentage Error = Decimal Form of Relative Error x 100**.

Truncation error

A truncation error is the difference between an actual and a truncated, or cut-off, value.

A truncated quantity is represented by a numeral with a fixed number of allowed digits, with any excess digits chopped off -- hence, the expression *truncated*

Example:

Consider the speed of light in a vacuum. The official value is 299,792,458 meters per second (m/s). In scientific (power-of-10) notation, it is expressed as 2.99792458 x 108 m/s. But truncating it to only two decimal places yields 2.99 x 108 m/s.

Since the truncation error is the difference between the actual value and the truncated value, in this case, it comes to the following:

2.99792458 x 108 - 2.99 x 108 = 0.00792458 x 108 m/s

Chopping Error:

- a type of **round-off error**
- truncated or chopping the last digit or last k digit of a rounding value

(IT"S NOT A TURNCATION ERROR)

Transcendental Functions

The transcendental function can be defined as a function that is **not algebraic** and **cannot be expressed in terms of a finite sequence** of algebraic operations such as sin x.

Keyword: **function which output is an infinite sequence**

Transcendental equation

A transcendental equation is an equation **into which transcendental functions** (such as *exponential, logarithmic, trigonometric, or inverse trigonometric*) of one of the variables (s) have been solved for.

A transcendental equation is an equation over the real (or complex) numbers that is not algebraic, that is, if at least one of its sides describes a transcendental function. [1]

Keyword: **the equation which contains transcendental function**

Characteristics (NOT SURE)

- non-algebraic
- infinite
- contains transcendental functions

(learn more : https://www.britannica.com/science/transcendental-function)

(If you are more interested about Errors, learn from : https://graphics.stanford.edu/courses/cs205a-13-fall/assets/notes/chapter1.pdf . Anyways, I haven't read the PDF yet. All of them are collected from different articles.)

Bisection Method

The bisection method is an approximation method to find the roots of the given equation by **repeatedly dividing the interval**. This method will divide the interval until the resulting interval is found, which is extremely small.

More:

- used to find the roots of a polynomial equation
- based on intermediate theorem on continuous function
- work by narrowing the gap between positive and negative interval until closes in on the correct answer

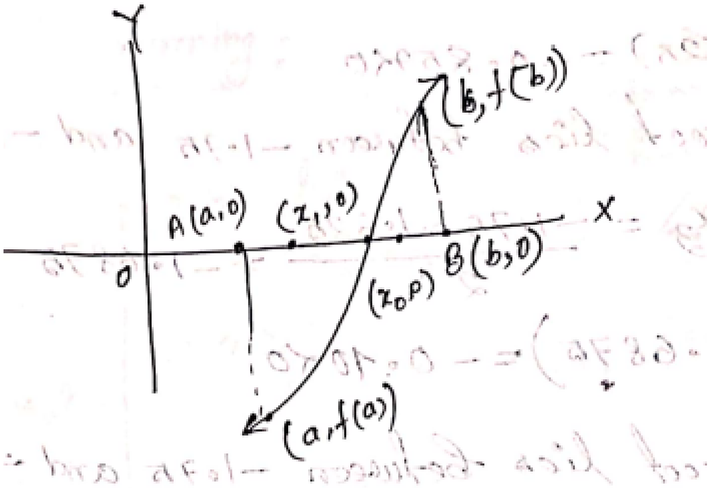
Theory & Proof: (From **Rupa)**

Theorem: if $f(x)$ be continuous ~~at~~ in $a \le x \le b$ and if $f(a)$ and $f(b)$ are opposite signs; then there exists at least one root of $f(x)=0$, say x_0, $f(x_0)=0$, $a<x_0<b$.

proof: Let $f(a)$ be negative and $f(b)$ be positive in the interval $[a,b]$. Then at least one root of the equation $f(x)$ lies in $[a,b]$. The root be $x_0 = \frac{1}{2}(a+b)$, which is obtained by dividing the distance between the points $A(A,0)$ & $B(b,0)$ into equal parts. ~~If f(x~~ if $f(x_0)=0$, then x_0 is the root of the given equations. otherwise the root will lie in the interval $[a,x_0]$ or $[x_0,b]$ according as $f(x_0)$ is positive or negative.

let $f(x_0)$ be positive the as before we devide the interval $[a,x_0]$ two equal parts

lets, $x_1 = \frac{1}{2}(a+x_0)$



Iterative Method

Iteration method:

If an equation $f(x)=0$ can be written in the form $x=\varphi(x)$ where $|\varphi'(x)|<1$.

$x_1=\varphi(x_0)$

$x_2=\varphi(x_1)$

$x_3=\varphi(x_2)$

$- - - - - -$

$- - - - - -$

$x_n=\varphi(x_{n-1})$

$x_{n+1}=\varphi(x_n)$

$f(x)=x^3+x^2-1=0$

$\Rightarrow x^2(x+1)=1$

$\therefore x=\dfrac{1}{\sqrt{x+1}}=\varphi(x)$

$\varphi'(x)=-\frac{1}{2}\cdot\dfrac{1}{(1+x)^{3/2}}$

$x_0=1$

$x_1=\dfrac{1}{\sqrt{0+1}}=1$

$x_2=\varphi(x_1)=\dfrac{1}{\sqrt{1+1}}=0.7071$

$x_3=\varphi(x_2)=\dfrac{1}{\sqrt{1+0.7071}}=0.76697$

$x_4=\varphi(x_3)=\dfrac{1}{\sqrt{1+0.76697}}=0.75229$

$x_5 = \varphi(x_4) = 0.75176$

$x_6 = \varphi(x_5) = 0.75187$

$x_7 = \varphi(x_6) = 0.75188$

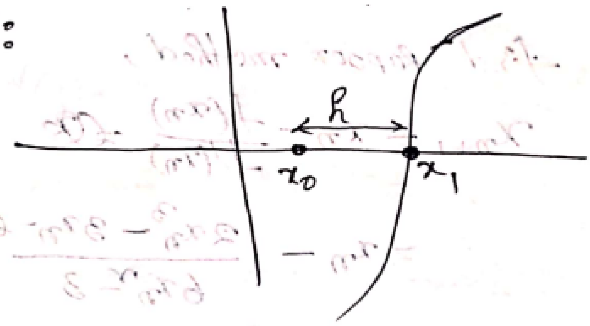
$x_8 = \varphi(x_7) = 0.75188$, Due to repetition of x_7 and x_8

$\therefore$ the required root is $= 0.75188$

Newton Rapson Method

Theory & Proof : (from Rupa)

Newton Rapson method:



~~Geernd~~

$x_1 = x_0 + h$

$f(x_1) = 0$

$f(x_0 + h) = 0$

according to tylor series,

$f(x_0) + h f'(x_0) + \frac{h^2}{2!} f''(x_0) + \frac{h^3}{3!} f'''(x_0) + \cdots = 0$

$\Rightarrow f(x_0 + h) = f(x_0) + h f'(x_0)$

$\Rightarrow f(x_0) + h f'(x_0) = 0$

$\therefore h = -\frac{f(x_0)}{f'(x_0)}$

$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \Rightarrow x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x) = 2x^3 - 3x - 6$

$f'(x) = 6x^2 - 3$

$f(1) = -7 < 0$; $f(2) = 4 > 0$

At least one root lies between 1 and 2.

we find Rapson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{2x_n^3 - 3x_n - 6}{6x_n^2 - 3}$$

$$x_{n+1} = \frac{4x_n^3 + 6}{6x_n^2 - 3} \quad \text{--- (1)}$$

$$x_0 = \frac{1+2}{2} = 1.5$$

$$\therefore x_1 = \frac{4(1.5)^3 + 6}{6 \times (1.5)^2 - 3} = 1.85714$$

$$x_2 = \frac{4 \times (1.85714)^3 + 6}{6 \times (1.85714)^2 - 3} = 1.78711$$

$$x_3 = \frac{4 \times (1.78711)^3 + 6}{6 \times (1.78711)^2 - 3} = 1.78378$$

$$x_4 = \frac{4 \times (1.78378)^3 + 6}{6 \times (1.78378)^2 - 3} = 1.78377$$

$$x_5 = \frac{4 \times (1.78377)^3 + 6}{6 \times (1.78377)^2 - 3} = 1.78377$$

precision < 0.001.

so, required root is 1.78377.

Shifting Operator

Shift Operator (E)

$$Ef(x) = f(x+h)$$
$$E^n f(x) = f(x+nh)$$
$$E^{-1} f(x) = f(x-h)$$
$$E^{-n} f(x) = f(x-nh)$$

$\triangle f(x) = f(x+h) - f(x)$ *[forward difference operator]*

$\nabla f(x) = f(x) - f(x-h)$ *[backward difference operator]*

Forward Difference Table

Construct Forward Difference Table : (from Rupa)

$$\nabla y_0 = y_1 - y_0 \qquad \nabla^2 y_0 = \nabla y_1 - \nabla y_0 \qquad \nabla^3 y_0 = \nabla^2 y_1 - \nabla^2 y_0$$

$$\nabla y_1 = y_2 - y_1 \qquad \nabla^2 y_1 = \nabla y_2 - \nabla y_1 \qquad \nabla^3 y_1 = \nabla^2 y_2 - \nabla^2 y_1$$

$$\nabla y_2 = y_3 - y_2 \qquad \nabla^2 y_{n-1} = \nabla y_n - \nabla y_{n-1} \qquad \nabla^3 y_{n-1} = \nabla^2 y_n - \nabla^2 y_{n-1}$$

$$\nabla y_{n-1} = y_n - y_{n-1}$$

Ex: x: 0 5 10 15 20 25

$y = f(x)$: 7 11 14 18 24 32 (Forward dif

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
0	7					
5	11	4				
10	14	3	-1			
15	18	4	1	2		
20	24	6	2	1	-1	
25	32	8	2	0	-1	

Interpolation

In short, interpolation is **a process of determining the unknown values that lie in between the known data points** x = 3   5   7   9   11

y= 2   6   8   12   13

The difference between x_i and x_{i+1} is equal. **2 for this example.**

if,

x= 3.5 (at starting) → we will use **Newton Forward Interpolation**

x= 5.1 - 7.9 → we will use **Central Difference Interpolation (Out of syllabus)**

x=9.5 (at end) → we will use **Newton Backward Interpolation**

Newton Forward Interpolation

Proof (from Rupa):

Theory: Newton's formula for forward interpolation with equal interval is

$$f(x+hu) = f(x) + u\Delta f(x) + \frac{u(u-1)}{2!}\Delta^2 f(x) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(a) + \cdots + \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n f(x).$$

proof:

Let $y = f(x)$ and $y_0, y_1, y_1 \cdots y_n$ are values corresponding to point $x_0, x_0+h, x_0+2h, \cdots, x_0+nh$.

suppose we find to $f(x) = y$ at point

$$x = x_0 + uh \left(u = \frac{x - x_0}{h}\right)$$

We know that by definition of E

$$E^u f(x) = f(x+uh)$$

$$\Rightarrow E^u f(x_0) = f(x_0 + uh)$$

$$\Rightarrow f(x_0+uh) = E^u f(x_0)$$

$$\therefore f(x) = y, \quad \Rightarrow f(x_0) = y_0$$

$$\Rightarrow E^u y_0 = f(x_0 + uh) \qquad \because E = 1 + \Delta$$

$$\Rightarrow f(x_0 + uh) = (1+\Delta)^u y_0 \qquad \because (1+x)^n = 1 + nx + \frac{n(n-1)}{2} + \cdots$$

$$\Rightarrow \cancel{[1 + p\Delta}$$

$$= \left[1 + u\Delta + \frac{u(u-1)}{2!}\Delta^2 + \frac{u(u-1)(u-2)}{3!}\Delta^3 + \cdots\right] y_0$$

$$= \cancel{[1 + n\Delta}$$

$$= \left[1 + u\Delta + \frac{u(u-1)}{2!}\Delta^2 + \frac{u(u-1)(u-2)}{3!}\Delta^3 + \cdots\right] f(x)$$

$$= f(x) + u\Delta f(x) + \frac{u(u-1)}{2!}\Delta^2 f(x) + \frac{u(u-1)(u-2)}{3!}\Delta^3 f(x) - - \cdots \frac{u(u-1)(u-2)\cdots(u-n+1)}{n!}\Delta^n f(x)$$

(proved)

Math : (from Rupa)

Procedure:

- Create a difference table
- Use the formula and calculate
  - h = difference b/w two contiguous x
  - f(a), f^2(a), …. → upper value (1st row)

x	y_0	Δy	$\Delta^2 y$	$\Delta^3 y$
20	0.312	0.0187	−0.0010	−0.0003
23	0.3907	0.0177	−0.0013	
26	0.1384	0.0164		
29	0.1818			

$x = 21$

Here, $u = \frac{x - x_0}{h}$

$$= \frac{21-20}{3} = 0.3333$$

according to Newton's forward formula,

$$y(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$= 0.312 + 0.3333 \times 0.0187 + \frac{0.3333(0.3333-1)}{2!}(-0.0010)$$

$$+ \frac{0.3333(0.3333-1)(0.3333-2)}{3!} \times (-0.0003)$$

$$= 0.3583$$

Newton Backward Interpolation

less important

- starting from below
- ‘+’ sign instead of ‘-’

(start taking the value of y from bottom → lower value)

Newton's Backward Difference formula
$p = \frac{x - x_n}{h}$ $y(x) = y_n + p\nabla y_n + \frac{p(p+1)}{2!} \cdot \nabla^2 y_n + \frac{p(p+1)(p+2)}{3!} \cdot \nabla^3 y_n + \frac{p(p+1)(p+2)(p+3)}{4!} \cdot \nabla^4 y_n + \ldots$

Missing Term in Interpolation

(YT: https://www.youtube.com/watch?v=P7fvPqdNOjM&ab_channel=B.K.TUTORIALS)

From Mishu:

Example :

x :	0	5	10	15
y :	7	?	18	?

Find missing term from the following

CamScanner

$\Delta^2 y_0 = 0$ | Here, $y_0 = f(x)$

$\Rightarrow (E-1)^2 y_0 =$

$\Rightarrow (E^2 - {}^2C_1 E^{2-1} + {}^2C_2 E^{2-2}) y_0 = 0$

$\Rightarrow (E^2 - 2E^1 + 1) f(x) = 0$

$E^n f(x) = f(x+nh)$

$\Rightarrow \cancel{f(x+2) - 2f(x+1) + f(x) = 0}$

$\Rightarrow f(x+2\times5) - 2f(x+1\times5) + f(x+0\times5) = 0$

$\Rightarrow f(x+10) - 2f(x+5) + f(x) = 0$ —— (i)

putting $x=0$ in (i)

$f(10) - 2f(5) + f(0) = 0$

$\Rightarrow 18 - 2f(5) + 7 = 0$

$\Rightarrow -2f(5) = -25$

$\Rightarrow f(5) = 12.5$

putting $x=5$ in (i)

$f(15) - 2f(10) + f(5) = 0$

$\Rightarrow f(15) - 34 + 12.5 = 0$

$\Rightarrow f(15) = 23.5$

Newton Divided Difference

$$f(x) = f(x_0) + (x-x_0)\, f(x_0, x_1) + (x-x_0)(x-x_1)\, f(x_0, x_1, x_2)$$
$$+ \dots + (x-x_0)(x-x_1)(x-x_2)\dots(x-x_{n-1})\, f(x_0, x_1, \dots, x_n)$$

Example

x : 4 5 7 10 11 13

$f(x)$: 48 100 294 900 1210 2028

$f(8)$

$f(15)$

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
4	48	$\frac{100-48}{5-4} = 52$	$\frac{97-52}{7-4} = 15$	$\frac{21-15}{10-4} = 1$	$\frac{1-1}{11-4} = 0$	0
5	100	$\frac{294-100}{7-5} = 97$	$\frac{202-97}{10-5} = 21$	$\frac{27-21}{11-5} = 1$	$\frac{1-1}{13-5} = 0$	
7	294	$\frac{900-294}{10-7} = 202$	$\frac{310-202}{11-7} = 27$	$\frac{33-27}{13-7} = 1$		
10	900	$\frac{1210-900}{11-10} = 310$	$\frac{409-310}{13-10} = 33$			
11	1210	$\frac{2028-1210}{13-11} = 409$				
13	2028					

Using Newton's interpolation for unequal interval

$$f(8) = 48 + (8-4) \times 52 + (8-4)(8-5) \times 15 + (8-4)(8-5)(8-7) \times 1$$
$$= 448$$

$$f(15) = 48 + (15-4) \times 52 + (15-4)(15-5) \times 15 + (15-4)(15-5)(15-7) \times 1$$
$$= 3150$$

What is Curve Fitting ?

(mark 4)

Curve fitting is ***the process of constructing a curve, or mathematical function, that has the best fit to a series of data points***

Advantages:

- Simplicity: It is very easy to explain and to understand
- Applicability: There are hardly any applications where least squares doesn't make sense
- Theoretical Underpinning: It is the maximum-likelihood solution and, if the Gauss-Markov conditions apply, the best linear unbiased estimator

Disadvantages/Drawbacks:***

- **Sensitivity to outliers**
- Test statistics might be ***unreliable*** when the data is not normally distributed (but with many datapoints that problem gets mitigated)
- Tendency to **overfit** data (LASSO or Ridge Regression might be advantageous)
- It can be quite **sensitive to the choice of starting values.**
- It is **not readily applicable to censored data**

Numerical Integration

Numerical Integration is a process of evaluating or obtaining a definite integral from a set of numerical values of the integrand f(x).

$h = \frac{b-a}{n}$

n = stripe, (generally, it's 6. why 6 ? ⇒ because 6 is divisible by 2 and 3 ? Why 2 & 3 ? Check below ?)

Trapezoidal Rule

$\int_a^b f(x)dx = h(\frac{y_0+y_n}{2} + y_1.... + y_n)$

Applicable for any no. interval.

Simpson one-third rule

$\int_a^b f(x)dx = \frac{h}{3}(y_0 + y_n + 4(y_1 + y_3 + ...) + 2(y_2 + y_4 + ...))$

Applicable for only **even intervals.**

Simpson three-by-eight rule

$\int_a^b f(x)dx = \frac{3h}{8}(y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5...) + 2(y_3 + y_6 + ...))$

Applicable for only **multiple of 3 intervals.**

WHY 6 ? ⇒ Because 6 is an **even** ∩ **multiple of 3**. And it's the minimum which fill-up both conditions.

Lagrange Interpolation

The **Lagrange interpolation** formula is a way to find a polynomial which takes on certain values at arbitrary points

* Apply inverse lagrange's method to find the value of x when $f(x) = 15$ from the given data.

x:	5	6	4	11
$f(x)$:	12	13	14	16

$\Rightarrow$ Given: $y = 15$ to find x,

$x_0 = 5$, $x_1 = 6$, $x_2 = 4$, $x_3 = 11$

$y_0 = 12$, $y_1 = 13$, $y_2 = 14$, $y_3 = 16$

By inverse lagrange's method,

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)}x_0 + \frac{(y-y_0)(y-y_2)(y-y_3)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}x_1 + \frac{(y-y_0)(y-y_1)(y-y_3)}{(y_2-y_0)(y_2-y_1)(y_2-y_3)}x_2 + \frac{(y-y_0)(y-y_1)(y-y_2)}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}x_3$$

= ....

= $\boxed{}$

* Use Newton's divided formula to find $f(x)$ given,

x:	0	2	3	6
$f(x)$:	648	704	729	792

also find $f(4)$ and $f'(4)$

Solving the equation for y and removing common factor

$$y = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} * y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})} \times y_n$$

This equation is called Lagrange's equation for interpolation.

Approaches to Prove Lagrange:

* Way to prove Lagrange

1) ~~f(x)~~ . . . x_0 x_1 x_2 x_3 . . . x_n

y_0 y_1 y_2 y_3 . . . y_n

2) $f(x_1, x_0, \ldots x_n) = 0$ . . . (i)

3) $f(x_1, x_0 \cdots x_n) = \frac{f(x)}{(x-x_0)\cdots(x-x_n)} + \cdots$

$\Rightarrow 0 = \frac{f(x)}{(x-x_0)\ldots(x-x_n)} + \cdots$

4) Transposing all except first to the right side

$$\frac{y}{(x-x_0)\cdots(x-x_n)} = \frac{y_0}{(x_0-\cancel{x_0})(x_0-x_1)} + \cdots$$

5) Solving the eqn for y,

$$y = \frac{(x-x_1)\cdots(x-x_n)}{(x_0-x)(x-x_n)} y_0 + \cdots$$

(Prove).

LU Decomposition Factorization

Example 7.2 : Factorize the matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$\Rightarrow$

Let, $A = LU$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{11} l_{21} & u_{12} l_{21} + u_{22} & u_{13} l_{21} + u_{23} \\ u_{11} l_{31} & u_{12} l_{31} + u_{22} l_{32} & u_{13} l_{31} + u_{23} l_{32} + u_{33} \end{bmatrix}$$

By equating both side (in column)

$u_{11} = 2$

$u_{11} l_{21} = 1 \quad \Rightarrow l_{21} = \frac{1}{2}$

$u_{11} l_{31} = 3 \quad \Rightarrow l_{31} = \frac{3}{2}$

$u_{12} = 3$

$u_{12} l_{21} + u_{22} = 2 \quad \Rightarrow u_{22} = 2 - \frac{3}{2} = \frac{1}{2}$

$u_{12} l_{31} + u_{22} l_{32} = 1 \Rightarrow u_{22} l_{32} = 1 - \frac{9}{2} = \frac{-7}{2}$

$\Rightarrow l_{32} = -7$

$u_{13} = 1$

$u_{13} l_{21} + u_{23} = 3 \quad \Rightarrow u_{23} = 3 - \frac{1}{2} = \frac{5}{2}$

$u_{13} l_{31} + u_{23} l_{32} + u_{33} = 2$

$\Rightarrow 1 \cdot \frac{3}{2} + \frac{5}{2} \cdot (-7) + u_{33} = 2$

$\Rightarrow \quad u_{33} = 2 - \frac{3}{2} + \frac{35}{2}$

$= \frac{4 - 3 + 35}{2} = \frac{36}{2}$

$= 18$

CamScanner

$$\therefore L = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ 3/2 & -7 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{bmatrix}$$

To solve the Equation using LU Decomposition:

LU Decomposition:

1) $L = \begin{vmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{vmatrix}$ $\quad U = \begin{vmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{vmatrix}$

2) $A = LU$

3) $Ly = B$

1) $Ux = y$

Least Square Method

([https://byjus.com/maths/least-square-method/#:~:text=The least square method is,the points from the curve](https://byjus.com/maths/least-square-method/#:~:text=The%20least%20square%20method%20is,the%20points%20from%20the%20curve).)

The **least square method** is the process of finding the **best-fitting curve** or line of **best fit for a set of data points** by reducing the sum of the squares of the points from the curve.

The equation of least square line is given by **Y = a + bX**

Normal equation for 'a':

∑Y = na + b∑X

Normal equation for 'b':

∑XY = a∑X + b∑X2

Question 1.: Find a straight line that fits the following data

x_i	8	3	2	10	11	3	6	5	6	8
y_i	4	12	1	12	9	4	9	6	1	14

Solution:

Straight line equation is y = a + bx.

$n=10$

The normal equations are

∑y = an + b∑x

∑xy = a∑x + b∑x^2

x	y	x^2	xy
8	4	64	32
3	12	9	36
2	1	4	2
10	12	100	120
11	9	121	99
3	4	9	12
6	9	36	54
5	6	25	30
6	1	36	6
8	14	64	112
∑x = 62	∑y = 72	∑x^2 = 468	∑xy = 503

Substituting these values in the normal equations,

10a + 62b = 72….(1)

62a + 468b = 503….(2)

(1) × 62 – (2) × 10,

620a + 3844b – (620a + 4680b) = 4464 – 5030

-836b = -566

b = 566/836

b = 283/418

b = 0.677

Substituting b = 0.677 in equation (1),

10a + 62(0.677) = 72

10a + 41.974 = 72

10a = 72 – 41.974

10a = 30.026

a = 30.026/10

a = 3.0026

Therefore, the equation becomes,

y = a + bx

y = 3.0026 + 0.677x

Question 2 :

- y=something-something convert it to y=a+bx, and find new A, X, Y
- Create table
- Use those two formula and get the value of A,b … convert A to a.

From Fayaz:

9.(b)

$$y = ax^b$$

$$\log y = \log a + b \log x$$

$$Y = A + BX$$

$$\sum Y = nA + B\sum X$$

$$\sum XY = A\sum X + B\sum X^2$$

Let,

$\log y = Y$

$\log a = A$

$b = B$

$\log x = X$

$\boxed{n = 4}$

x	y	$X = \log_e x$	$Y = \log_e y$	XY	X^2
1	5	0	1.60944	0	0
2	7	0.69315	1.94591	1.34881	0.48046
3	9	1.09861	2.19722	2.41389	1.20694
4	10	1.38629	2.30258	3.19209	1.9218

$\sum X = 3.17805$ $\quad \sum Y = 8.05515$ $\quad \sum XY = 6.95479$ $\quad \sum X^2 = 3.6092$

$$8.05515 = 4A + B \times 3.17805 \quad \text{.......(i)}$$

$$6.95479 = A \times 3.17805 + B \times 3.6092 \quad \text{.......(ii)}$$

$$A = 1.60721$$

$$B = 0.51173$$

$\log a = A$

$a = e^A$

$= 4.98887$

$b = B$

$= 0.51173$

$\therefore y = (4.98887)\, x^{(0.51173)}$

Euler Method:

(https://www.youtube.com/watch?v=u5ggAyOOTUw&t=19s)

The Euler's method is a first-order numerical procedure for **solving ordinary differential equations** (ODE) with a given initial value.

$$n = \frac{b - x_0}{h}$$

$$y_{i+1} = y_i + hf(x_i, y_i)$$

where,

- y_{i+1} is the next estimated solution value;
- y_i is the current value;
- h is the interval between steps;
- $f(x_i, y_i)$ is the value of the derivative at the current (x_i, y_i) point.

Euler proof (Approach) :

$$\tan\theta \approx \frac{\Delta y}{\Delta x}$$

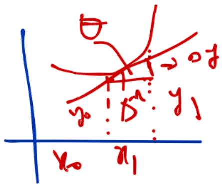
$$\frac{dy}{dx} = f(x,y)$$

$$\Rightarrow \Delta y \approx \Delta x \tan\theta$$

$$\Rightarrow y_1 - y_0 \approx \Delta x \left(\frac{dy}{dx}\right)_0$$

$$\Rightarrow y_1 \approx y_0 + \Delta x\, f(x_0, y_0)$$

$$\Rightarrow y_1 = y_0 + h f(x_0, y_0)$$



Modified Euler Method:

(https://www.youtube.com/watch?v=xLGDGeFZTnQ)

Instead of approximating f(x, y) by as in Euler's method. In the Modified Euler Method: we have the iteration formula

$$y_1^{(0)} = y_0 + hf(x_0, y_0)$$

$$y_1^{(n+1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(n)})] \ , n = 0,1,2 \dots \dots$$

Take $n = 1$ to solve it by one step.

Accuracy

Euler method has a **truncation error.** To solve this problem modified Euler method is introduced. How ? →

- **It takes arithmetic average of an interval (Xi, Xi+1) instead of a poin**t. bla bla bla….

Range Kutta Method

(https://www.youtube.com/watch?v=fll1HdYy6vk&t=696s - 2nd Order)

(https://www.youtube.com/watch?v=JhI6cLRjKHY - 4th Order)

Runge–Kutta method is **an effective and widely used method for solving the initial-value problems of differential equations**. Runge–Kutta method can be used to construct high order accurate numerical method by functions' self without needing the high order derivatives of functions.

4th Order from Mishu:

Therefore,

Fourth order of Runge-kutta method,

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

when,

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}k_1\right)$$

$$k_3 = hf\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{1}{2}k_2\right)$$

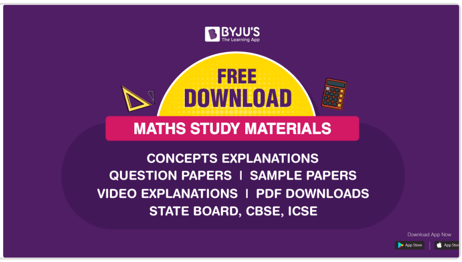
$$k_4 = hf(x_0 + h,\ y_0 + k_3)$$

Gauss Seidel Method

The Jacobi and Gauss-Seidel Iterative Methods

Iterative methods Jacobi and Gauss-Seidel in numerical analysis are based on the idea of successive approximations. This iterative method begins with one or two initial

https://byjus.com/maths/iterative-methods-gauss-seidel-and-jacobi/



Solve the system of equations using the Gauss-Seidel Method

$45x_1 + 2x_2 + 3x_3 = 58$

$-3x_1 + 22x_2 + 2x_3 = 47$

$5x_1 + x_2 + 20x_3 = 67$

Obtain the result correct to three decimal places.

Solution:

First, check for the convergence of approximations,

$45 > 2 + 3$

$22 > -3 + 2$

$20 > 5 + 1$

Hence, the given system of equations are strongly diagonally dominant, which ensures the convergence of approximations. Let us take the initial approximation, $x_1^{(0)} = 0$, $x_2^{(0)} = 0$ and

$x_3^{(0)} = 0$

First Iteration:

$x_1^{(1)} = 1/45[58 - 2 \times 0 - 3 \times 0\,] = 1.28889$

$x_2^{(1)} = 1/22[\,47 + 3 \times 1.28889 - 2 \times 0\,] = 2.31212$

$x_3^{(1)} = 1/20[67 - 5 \times 1.28889 - 1 \times 2.31212] = 2.91217.$

Second Iteration:

$x_1^{(2)} = 1/45[58 - 2 \times 2.31212 - 3 \times 2.91217\,] = 0.99198$

$x_2^{(2)} = 1/22[\,47 + 3 \times 0.99198 - 2 \times 2.91217\,] = 2.00689$

$x_3^{(2)} = 1/20[67 - 5 \times 0.99198 - 1 \times 2.00689] = 3.00166.$

Likewise there will be modification in approximation with each iteration.

kth iteration	0	1	2	3	4
x_1	0.000	1.28889	0.99198	0.99958	1.0000
x_2	0.000	2.31212	2.00689	1.99979	1.99999
x_3	0.000	2.91217	3.00166	3.00012	3.00000

After the fourth iteration, we get $|x_1^{(4)} - x_1^{(3)}| = |1.0000 - 0.99958| = 0.00042$

$|x_2^{(4)} - x_2^{(3)}| = |1.99999 + 1.99979| = 0.00020$

$|x_3^{(4)} - x_3^{(3)}| = |\,3.0000 - 3.00012| = 0.00012$

Since, all the errors in magnitude are less than 0.0005, the required solution is

$x_1 = 1.0,\ x_2 = 1.99999,\ x_3 = 3.0$

Preferred Initial value, x1=x2=x3=0

Numerical Differentiation:

- Create Forward Difference Table
- Use the formulae for 1st, 2nd and 3rd derivative

$$* \; f(a+xh) = f(a) + x\Delta f(a) + \frac{x(x-1)}{2!}\Delta^2 f(a) + \frac{x(x-1)(x-2)}{3!}\Delta^3 f(a) + \dots$$

$$= f(a) + x\Delta f(a) + \frac{x^2-x}{2!}\Delta^2 f(a) + \frac{x^3-2x^2-x^2+2x}{3!}\Delta^3 f(a) + \dots$$

$\Rightarrow$ 1st derivation : $f'(x) = \frac{1}{h}\left[\Delta f(a) + x\Delta^2 f(a) - \frac{1}{2}\Delta^2 f(a) + \frac{1}{2}x^2\Delta^3 f(a) - x\Delta^3 f(a) + \frac{2}{6}\Delta^3 f(a) + \dots\right]$

$\Rightarrow$ 2nd derivation : $f''(x) = \frac{1}{h^2}\left[\Delta^2 f(a) + x\Delta^3 f(a) - \Delta^3 f(a)\right]$

$\Rightarrow$ 3rd derivation : $f'''(x) = \frac{1}{h^3}\left[+\ \Delta^3 f(a)\right]$

$a + xh = \square \rightarrow$ যার value বের করব / $x = \square$

$\Rightarrow x = \square$

Different between Gauss Elimination Method & Gauss Jordan Method

(https://www.geeksforgeeks.org/difference-between-gauss-elimination-method-and-gauss-jordan-method-numerical-method/)

In mathematics, the Gaussian elimination method is known as the row reduction algorithm for solving linear equations systems.

Gauss Elimination Method	Gauss Jordan Method
upper triangular system	reduces to diagonal matrix
For large systems, Gauss Elimination Method is not preferred.	For large systems, Gauss Jordan Method is preferred to Gauss Elimination Method
It does not seem to be easier	It seems to be easier
it requires about 50 percent fewer operation than	requires about 50 percent more operations than Gauss elimination Method.

EXTRA:

Divided Difference Proof Idea and finding nth Divided Difference

Divided Difference

* Way to prove divided difference

* Finding Divided Difference

1st, $f(a,b) = \dfrac{f(b)-f(a)}{b-a}$

$= \dfrac{f(b)}{b-a} + \dfrac{f(a)}{a-b}$

2nd, $f(a,b,c) = \dfrac{f(b,c) - f(a,b)}{c-a}$

$= \dfrac{1}{c-a}\left[\cancel{f}\ f(b,c) - f(a,b)\right]$

$= \dfrac{1}{c-a}\left[\dfrac{f(b)}{b-c} + \dfrac{f(c)}{c-b} - \dfrac{f(a)}{a-b} - \dfrac{f(b)}{b-a}\right]$

$= \dfrac{f(a)}{(a-c)(a-b)} + \dfrac{f(c)}{(c-a)(c-b)} + \dfrac{f(b)}{\cancel{(b-b)}(b-c)(b-a)}$

3rd, $f(a,b,c,d) = \dfrac{f(b,c,d) - f(a,b,c)}{d-a}$

$= \boxed{\quad}$

if f(x) is given, then find the value of f(a), f(b) and solve this

Taylor:

(https://www.youtube.com/watch?v=82IDoaiYU0c&ab_channel=MKSTUTORIALSbyManojSir)

* Taylor Method

$$y_{n+1} = y_n + h y'_n + \frac{h^2}{2!} y''_n + \frac{h^3}{3!} y'''_n + \cdots$$

where, $h = x - x_0$

Problem $\frac{dy}{dx} = x + y$. $y(1) = 0$. $x = 1.2$,

– Partial differentiation $h = 0.1$

$y' = x + y$ $y'_0 = 1$

$y'' = 1 + y'$ $y''_0 = 1 + 1 = 2$

$y''' = y''$ $y'''_0 = 2$

$y^{IV} = y'''$ $y^{IV}_0 = 2$

$$y_1 = y(x_1) = y_0 + h y'_0 + \frac{h^2}{2!} y''_0 + \frac{h^3}{3!} y''' + \cdots$$

$$y_1 = y(1.1) = 0 + 0.1 \times 1 + \frac{(0.1)^2}{2!} \times 2 + \frac{(0.1)^3}{3!} + \cdots$$

$$= 0.11034 \Rightarrow 0.1103 \ (4\text{th})$$

Now, $x_1 = 1.1$, $y_1 = 0.1103$.

$x_2 = x_1 + h = 1.2$

$y'_1 = x_1 + y_1 = 1.1 + 0.1103 = 1.2103$

$y''_1 = 1 + y'_1 = 1 + 1.2103 = 2.2103$

$y'''_1 = 0 + y''_1 = 2.2103$.

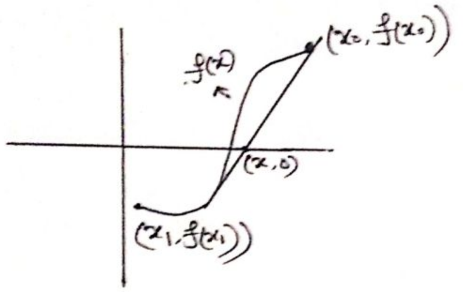
$$y_2 = y(1.2) = y_1 + h y'_1 + \frac{h^2}{2!} y''_1 + \frac{h^3}{3!} y''' + \cdots$$

$$= \boxed{}$$

ASH2101008M

Some interesting way to prove :

* Newton Raphson

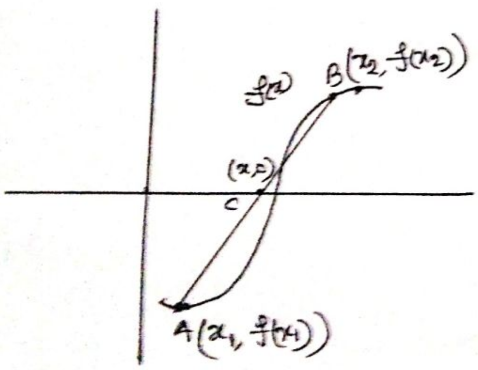


Equation of tangent,

$$y - y_0 = f'(x_0)(x - x_0)$$

$$\Rightarrow x = x_0 - \frac{f(x_0)}{f'(x_0)} \quad [y = 0]$$

* Secant & Regula Falsi



C is intermediate point.

Slope AB = Slope CB

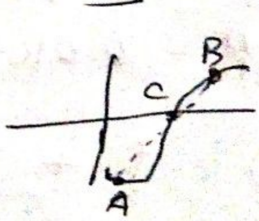
$$\Rightarrow \frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{0 - f(x_2)}{x - x_2}$$

$$\Rightarrow x = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

*

$$x = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \quad \begin{bmatrix} x_0 \to + \\ x_1 \to - \end{bmatrix}$$

Regula falsi : $-x \to x_1 = x$
$+x \to x_0 = x$



Secant : $x_0 = x_1$
$x_1 = x$